MATH 3XXX- Lecture Notes I

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Definition according to Aristotle

Aristotle (384-322 BC) introduced the concept of definition. According to Aristotle a species is defined by giving its *genus proximus* (nearest genus) and its *differentia specifica* (specific difference). The differentia is an attribute or set of attributes which unambiguously distinguishes members of species to be defined from other members of the same genus. An example is worth a thousand words. The following is a definition *per genus proximus et differentia specifica* of the species *human*:

Definition. An animal is *human* if it has the ability to reason.

The species *human* is defined by specifying the genus (animal) and the differentia specifica (the capacity to reason).

In Chemistry we might define the term *exothermic* as follows:

Definition. A reaction is *exothermic* if it liberates heat.

The above is a definition *per genus proximus et differentia specifica* of the species of chemical reaction called exothermic. The genus here is the set of reactions and the differentia specifica the attribute "liberates heat".

Definition in Mathematics

Mathematical definition conforms with Aristotle's format. Consider the definition of the term *even*.

Definition. An integer n is *even* if there exists an integer k such that $n = 2k$.

In this example, we define the set of *even numbers* (the species). The role of genus proximus is played by the set of integers. The differentia specifica is expressed by the predicate, "there exists an integer k such that $n = 2k$". This predicate specifies the subset of the set of integers which we call even.

It is convention in Mathematical writing to place the species to be defined in italics or underline it when writing by hand. Students will be penalized for ignoring this convention.

The differentia is often a conjuntion of a number of predicates. A good example of this is the definition of the concept of a relation.

Definition. A triple (A, B, G) is a *relation* if

(F1.) A is a set,
(F2.) B is a set,
(F3.) G is a subset of $A \times B$.

In this example we define the species *relation* by specifying its genus (the class of triples) and differentia (the conjunction of the three predicates (F1-F3))

Exercises

1. Give definitions *per genus proximus et differentia specifica* of the following concepts.

   (a) *Elephant.*

   (b) *Triangle.*

   (c) *Maximum* of a subset of the natural numbers.

   (d) *Sequence.*

   (e) *Predicate*

2. Modify the definition of *relation* given above so as to obtain a definition of *function.*

3. Explain what (if anything) is wrong with each of the following "definitions":

   (a) *A circle* is a figure all of whose points are equidistant from a given point.

   (b) An *elephant* is an animal with four feet.

   (c) A man is a *dentist* if he practices dentistry.

   (d) A *thief* is a man who steals money.

4. Look up the concept of *virtue* in Aristotle's works on Ethics. Identify the genus and differentia.

5. In Plato's Meno how does Socrates explain what is wrong with Meno's definition, "virtue is the capacity to rule"?