

Math 3018 - Philip Penance <http://penance.us/> - Lecture Summary - On Statements and Truth

A sentence which possesses a truth value will be called a *statement*. However the conception of truth is complicated by the fact that there are different types of truth and a statement may be true or false according to which type of truth applies. Consider for example the following statements:

1. There is no fire without oxygen.
2. The triangle has three sides.
3. $\sqrt{2}^{\sqrt{2}}$ is either rational or irrational.
4. Given a line L , and a point P not on L , there is exactly one line through the point P parallel to the line L .
5. The sum of the angles in any triangle is 180 degrees.
6. The sum of the angles in any triangle is not 180 degrees.
7. Every even integer greater than 2 can be written as the sum of two primes.
8. The triangle does not have three sides.

For early man [1] was an empirical truth. For the scientist of today its truth would depend on the definition of fire as a chemical reaction involving oxygen or perhaps a more complicated definition. [2] is true by definition of a triangle. We will see later that [3] is a logical truth (tautology). In addition to primitive terms which we do not define, mathematical and physical theories contain statements which we do not attempt to prove but accept as true. Such statements are called *axioms*. [4] is an axiom of, and hence considered true in Euclidean geometry. It may be false in other geometries. Statements deduced from axioms by a mathematical proof are called *theorems*. [5] is a theorem of, and hence true in Euclidean geometry. In Euclidean geometry [6] is false. [7] is a conjecture of Goldbach (1742). Since it is so far unproven its truth is unknown. [8] is the *negation* of statement [2]. Given any statement p its *negation* (denoted $\neg p$) is the statement which is true if p is false and false if p is true. Since [2] is true, its negation [8] is false.

We see that mathematical truth is not the same as empirical truth. Mathematical statements cannot be proven by experiment in the real world (say by measuring the angles of a triangular object or by the use of so called “manipulatives”).

Not everything that we say is a statement in the sense that a truth value can be assigned. Consider the set T of all true statements uttered in the Senate of Puerto Rico. A senator says: “*The statement that I am now uttering is not in T* ”. His utterance cannot be considered a statement since if it were true (in T) then it contradicts itself and, similarly, if it were false it also contradicts itself. Thus, no truth value can be assigned.