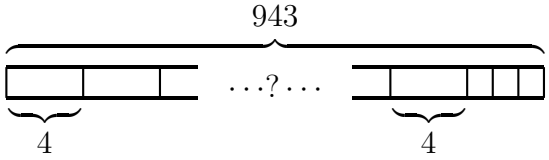
Math 3131 Prof. Pennance – Summary of Lecture 10 - A Division Algorithm

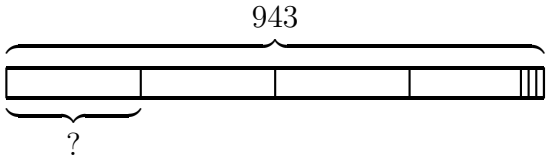
1. The division problem $943 \div 4 = 235\ R\ 3$.

2. Egyptian equivalent:
   $\mathcal{9}\ \mathcal{9}\ \mathcal{9}\ \mathcal{9}\ \mathcal{9}\ \mathcal{9}\ \mathcal{9}\ \mathcal{9}\ \mathcal{9}\ \cap\ \cap\ \cap\ \cap\ |\ |\ |$
   $\div\ |\ |\ |\ |\ \ =\ \mathcal{9}\ \mathcal{9}\ \cap\ \cap\ \cap\ |\ |\ |\ |\ |\ \ R\ |\ |\ |$

3. Measurement Interpretation: 235 is the number of groups when 943 is divided into groups of 4 and 3 remain.



4. Partitive Interpretation: 235 is the size of each group if 943 is divided into 4 equal groups and 3 remain.



5. Indo-Arabic and Egyptian versions of the standard division algorithm.

   Step 1. Indo-Arabic:

   Divide leftmost digit of dividend by 4

$$\begin{array}{r} 2 \\ 4\overline{)9} \\ \underline{8} \\ 1 \end{array}$$

   and so:

$$9 = 2 \times 4 + 1$$

   Step 1. Egyptian equivalent:

   Decompose the 9 $\mathcal{9}'s$ into 2 rows of 4 $\mathcal{9}$'s with remainder $\mathcal{9}$.

$$\mathcal{9}\mathcal{9}\mathcal{9}\mathcal{9}\mathcal{9}\mathcal{9}\mathcal{9}\mathcal{9}\mathcal{9} = \mathcal{9}\mathcal{9}\mathcal{9}\mathcal{9} + \mathcal{9}\mathcal{9}\mathcal{9}\mathcal{9} + \mathcal{9}$$

10^3	10^2	10^1	10^0
	$\mathcal{9}\ \mathcal{9}\ \mathcal{9}\ \mathcal{9}$ $\mathcal{9}\ \mathcal{9}\ \mathcal{9}\ \mathcal{9}$ $\mathcal{9} \mapsto$	$\cap\ \cap\ \cap\ \cap$	$\|\ \|\ \|$

   Step 2. Indo-Arabic:
   Calculate the next dividend (14) by multiplying the remainder (1) of previous step by 10 and adding to next digit (4) of dividend

   Step 2. Egyptian equivalent:
   Decompose the extra $\mathcal{9}$ into 10 $\cap$'s to obtain a total of 14 $\cap$'s.

10^3	10^2	10^1	10^0
	$\mathcal{9}\ \mathcal{9}\ \mathcal{9}\ \mathcal{9}$ $\mathcal{9}\ \mathcal{9}\ \mathcal{9}\ \mathcal{9}$	$\cap\ \cap\ \cap\ \cap\ \cap$ $\cap\ \cap\ \cap\ \cap\ \cap$ $\cap\ \cap\ \cap\ \cap$	$\|\ \|\ \|$

   Step 3. Indo-Arabic:

   Divide new dividend 14 by 4

$$\begin{array}{r} 3 \\ 4\overline{)14} \\ \underline{12} \\ 2 \end{array}$$

   and so:

$$14 = 3 \times 4 + 2$$

   Step 3. Egyptian equivalent:

   Write the 14 $\cap$'s as 3 rows of 4 $\cap$'s and one row of 2 $\cap$'s.

10^3	10^2	10^1	10^0
	$\mathcal{9}\ \mathcal{9}\ \mathcal{9}\ \mathcal{9}$ $\mathcal{9}\ \mathcal{9}\ \mathcal{9}\ \mathcal{9}$	$\cap\ \cap\ \cap\ \cap$ $\cap\ \cap\ \cap\ \cap$ $\cap\ \cap\ \cap\ \cap$ $\cap\ \cap \mapsto$	$\|\ \|\ \|$

   Step 4. Indo-Arabic

   Calculate the next dividend (23) by multiplying remainder (2) of previous step by 10 and adding the next digit (3) of dividend

   Step 4. Egyptian equivalent:

   Decompose the extra 2 $\cap$'s into 20 $|$'s to obtain a total of 23 $|$'s.

10^3	10^2	10^1	10^0
	𓎆 𓎆 𓎆 𓎆 𓎆 𓎆 𓎆 𓎆	∩ ∩ ∩ ∩ ∩ ∩ ∩ ∩ ∩ ∩ ∩ ∩	||||||||| ||||||||| |||||

Step 5. Indo-Arabic:

Divide new dividend 23 by 4:

$$\begin{array}{r} 5 \\ 4\overline{)23} \\ \underline{20} \\ 3 \end{array}$$

and so:

$$23 = 5 \times 4 + 3$$

Step 5. Egyptian equivalent:

Write the 23 𓏺's as 5 rows of 4 𓏺's and one row of 3 𓏺's.

10^3	10^2	10^1	10^0
	𓎆 𓎆 𓎆 𓎆 𓎆 𓎆 𓎆 𓎆	∩ ∩ ∩ ∩ ∩ ∩ ∩ ∩ ∩ ∩ ∩ ∩	|||| |||| |||| |||| |||| |||

Step 6. Indo-Arabic:

The previous steps are summarized in the table:

	10^2	10^1	10^0
Step	9	4	3
1	$2 \times 4 + 1$	4	3
2	2×4	14	3
3	2×4	$3 \times 4 + 2$	3
4	2×4	3×4	23
5	2×4	3×4	$5 \times 4 + 3$

From last row of table we see that

$$943 = (235 \times 4) + 3$$

and the quotient $q = 235$ and the remainder $r = 3$.

Step 6. Egyptian equivalent:

Counting rows of 4 in each denomination gives: $q =$ 𓎆𓎆∩∩∩\|\|\|\|\| and $r =$ \|\|\|.

6. Summary

The standard algorithm takes as "initial dividend" the leftmost digit of the actual dividend. The quotient and remainder are found when this initial dividend is divided by 4. If there are no more digits in the dividend the algorithm stops. Otherwise, a new dividend is calculated by multiplying the remainder from the previous step by 10 and adding to the next digit of the dividend. This cycle of activity is repeated starting with this new dividend and continues until no digits remain.

$$\begin{aligned} 9 &= \boxed{2} \times 4 + 1 \\ 14 &= \boxed{3} \times 4 + 2 \\ 23 &= \boxed{5} \times 4 + 3 \end{aligned}$$

7. Tabulation

$$\begin{array}{r} 235 \\ 4\overline{)943} \\ \underline{800} \\ 143 \\ \underline{120} \\ 23 \\ \underline{20} \\ 3 \end{array}$$

8. Another example $123456 \div 16$

$$\begin{aligned} 1 &= \boxed{0} \times 16 + 1 \\ 12 &= \boxed{0} \times 16 + 12 \\ 123 &= \boxed{7} \times 16 + 11 \\ 114 &= \boxed{7} \times 16 + 2 \\ 25 &= \boxed{1} \times 16 + 9 \\ 96 &= \boxed{6} \times 16 + 0 \end{aligned}$$

9. Tabulation

$$\begin{array}{r} 7716 \\ 16\overline{)123456} \\ \underline{112000} \\ 11456 \\ \underline{11200} \\ 256 \\ \underline{160} \\ 96 \\ \underline{96} \\ 0 \end{array}$$