Math 3131 Prof. Pennance –Summary of Lecture XV –Factors and Primes

1. A whole number $p > 1$ is *prime* if its only factors are 1 and p. A whole number $n > 2$ which is not prime is called *composite*.

2. Sieve of Eratosthenes

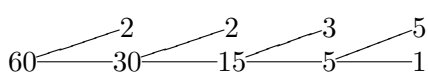
3. Let $n > 1$ be a whole number. Then its smallest factor $p > 1$ must be prime (and so n is a multiple of a prime.)

4. Example The complete list of factors of 35 is $\{1, 5, 7, 35\}$ The smallest factor greater than 1 is 5 which is prime. Notice 35 is a multiple of the prime 5.

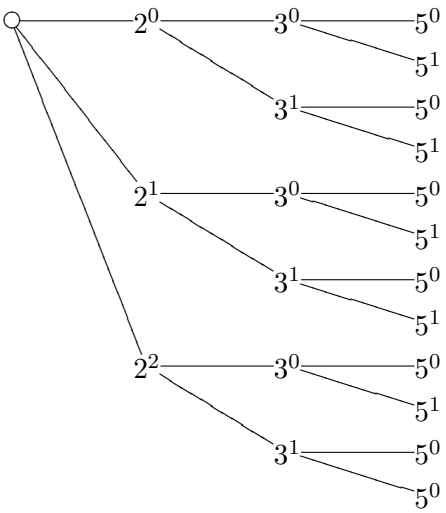
5. The Fundamental Theorem of Arithmetic: Every whole number $n > 1$ can be written as a product of primes in which the primes are unique (up to reordering).

6. Prime factorization of 60

$$\begin{aligned} 60 &= 2 \times 30 \\ &= 2 \times 2 \times 15 \\ &= 2 \times 2 \times 3 \times 5 \\ &= 2^2 \times 3^1 \times 5^1 \end{aligned}$$



7. All factors of 60



Each path from the root to a leaf corresponds to a factor. For example the path $2^2 \cdot 3^1 \cdot 5^0$ yields the factor 12. The complete list of factors obtained in this way is

$$\{1, 2, 3, 4, 5, 6, 10, 12, 15, 20, 30, 60\}$$

8. The *greatest common factor* or GCF of two integers n and m is the greatest whole number which is a factor of both n and m.

9. Example. Find $GCF(480, 216)$.
We find the prime factorization of each and match the primes which appear in both factorizations as follows.

$$\begin{array}{rcccccccccccc} 480 &=& 2 & \cdot & 2 & \cdot & 2 & \cdot & 3 & \cdot & 2 & \cdot & 2 \cdot 5 \\ & & | & & | & & | & & | & & & & \\ 216 &=& 2 & \cdot & 2 & \cdot & 2 & \cdot & 3 & \cdot & 3 & \cdot & 3 \end{array}$$

$GCF(480, 216) = 2 \cdot 2 \cdot 2 \cdot 3 = 24$

10. The *least common multiple* or LCM of two integers n and m is the least positive number which is a multiple of both n and m.

11. Example. Find $LCM(480, 216)$.

$$\begin{array}{rcccccccccccc} 480 &=& 2 & \cdot & 2 & \cdot & 2 & \cdot & 3 & \cdot & 2 & \cdot & 2 \cdot 5 \\ & & | & & | & & | & & | & & & & \\ 216 &=& 2 & \cdot & 2 & \cdot & 2 & \cdot & 3 & \cdot & 3 & \cdot & 3 \end{array}$$

$$LCM(480, 216) = \underbrace{2 \cdot 2 \cdot 2 \cdot 3}_{\text{One factor from each pair}} \cdot \underbrace{2 \cdot 2 \cdot 3 \cdot 3 \cdot 5}_{\text{Unpaired factors}} = 4,320$$