Math 3131 Prof. Pennance – Summary of Lecture 4 - Functions

1. A relation (A, B, R) is called a *function* if in the bipartite graph corresponding to the relation **every** vertex in A has degree **exactly** one.

2. Example: The associations:

   𓏺 ↦ 𓎆
   𓎆 ↦ 𓍢
   𓍢 ↦ 𓆼
   𓆼 ↦ 𓂭
   𓂭 ↦ 𓆐
   𓆐 ↦ 𓁨

   define a function (A, B, R) with domain A = {𓏺, 𓎆, 𓍢, 𓆼, 𓂭, 𓆐} and codomain B = {𓎆, 𓍢, 𓆼, 𓂭, 𓆐, 𓁨}. The graph R is the set of ordered pairs represented by the arrows above. The function f represents muliplication by 10 in ancient Egyptian.

3. A necessary and sufficient condition for a table to be the table of a function.

4. If $f = (A, B, R)$ is a function, then the unique vertex adjacent to a is denoted $f(a)$.

5. Let $f = (A, B, R)$ be the function defined in item 2 above. Then in the bipartite graph corresponding to f, 𓍢 is the unique vertex adjacent to 𓎆. Therefore we write f(𓎆) = 𓍢. The following notations are synonymous:

   (a) (𓎆, 𓍢) ∈ R.
   (b) f(𓎆) = 𓍢.
   (c) f : 𓎆 ↦ 𓍢.
   (d) 𓎆 $\overset{f}{\mapsto}$ 𓍢 .
   (e) f maps 𓎆 to 𓍢.
   (f) f associates 𓍢 with 𓎆.
   (g) 𓍢 is the image of 𓎆 under the function f.
   (h) (𓎆, 𓍢) is an edge in the bipartite graph corresponding to f.

6. $f(m/n) = m$ does not define a function.

7. A definition of the *equality* of functions and examples.

8. Some special functions

   (a) Constant functions.
   (b) Identity functions.
   (c) The successor function.
   (d) Cardinality.
   (e) Integer addition and corresponding set model.
   (f) Integer multiplication.
   (g) Sequences.
      i. Arithmetic sequences.
      ii. Geometric sequences.
   (h) The addition tables.
   (i) The multiplication tables.

9. The logical function (AND)

$$\begin{array}{rcl} (1,1) & \mapsto & 1 \\ (1,0) & \mapsto & 0 \\ (0,1) & \mapsto & 0 \\ (0,0) & \mapsto & 0 \end{array}$$

10. The logical function (OR)

$$\begin{array}{rcl} (1,1) & \mapsto & 1 \\ (1,0) & \mapsto & 1 \\ (0,1) & \mapsto & 1 \\ (0,0) & \mapsto & 0 \end{array}$$

11. Applications of AND and OR.