

## Math 3024 Prof. Pennance – Summary of Lectures on Trigonometry

1. Angles.
  - (a) A definition of *angle*.
  - (b) Angular measures.
  - (c) Arc length and area of a sector.
2. The function  $P : \mathbb{R} \rightarrow S^1$
3. Properties of  $P$ .
  - (a)  $P(t + 2\pi) = P(t)$
  - (b)  $P(t + \pi) = -P(t)$ .
  - (c)  $P(-t) = P(t)^*$ .
  - (d)  $d(P(0), P(s - t)) = d(P(s), P(t))$ .
4. The functions  $\sin$  and  $\cos$ .
5. Properties of  $\sin$  and  $\cos$ .
  - (a)  $\sin^2 + \cos^2 = 1$
  - (b) Parity of  $\sin$  and  $\cos$ .
  - (c) Periodicity of  $\sin$  and  $\cos$ .
  - (d)  $\cos(t \pm \pi) = -\cos t$   
 $\sin(t \pm \pi) = -\sin t$ .
  - (e)  $\cos(s - t) = \cos s \cos t + \sin s \sin t$
  - (f)  $\cos(s + t) = \cos s \cos t - \sin s \sin t$ .
  - (g)  $\cos(t - \pi/2) = \sin t$ .
6. Important values of  $\sin$  and  $\cos$ 
  - (a) Calculation of  $\cos \frac{\pi}{12}$  and  $\sin \frac{\pi}{12}$  using  
 $\cos(\frac{\pi}{12} + \frac{\pi}{12}) = \cos^2 \frac{\pi}{12} - \sin^2 \frac{\pi}{12}$ .  
 $1 = \cos^2 \frac{\pi}{12} + \sin^2 \frac{\pi}{12}$
  - (b) Calculation of  $\cos \frac{2\pi}{3}$  using formula for the cosine of a sum.
7. Example: Suppose  $P(\theta)$  lies on the line with equation  $y = -(4/3)x$  and that  $\pi/2 < \theta < \pi$ . Find:
  - (a)  $\sin \theta$ .
  - (b)  $\cos \theta$ .
  - (c)  $\tan \theta$ .
8. Graphs of  $\sin x$ ,  $\cos x$ .
9. Period, amplitude, and phase of a sinusoidal graph.
10. The effects of scaling, and translation on the fundamental period.
11. Example: graphs of:
  - (a)  $\sin 2x$ .
  - (b)  $\sin 2(x - \pi/4)$ .
  - (c)  $-3 \sin 2(x - \pi/4)$ .
  - (d)  $-3 \sin 2(x - \pi/4) + 2$ .
  - (e)  $-3 \sin(2x + \pi/2) + 2$ .
12. Given the graph  $G_f$  of  $f(t) = A \sin(\omega t - \phi)$  find values of  $A$ ,  $\omega$  and  $\phi$ .
13. Simple Harmonic Motion  $x = A \sin \omega t$ .
  - (a) Angular frequency ( $\omega$ ) – radians per second.
  - (b) Period – seconds/cycle.
  - (c) Frequency (1/period) – Hertz.
  - (d) Springs and Pendula
  - (e) Wave equation  $y = A \sin \omega(x - ct)$ .
14. The functions  $\sin x^\circ$  and  $\cos x^\circ$ .
15. Definitions and properties of  $\tan$ ,  $\cot$ ,  $\sec$ ,  $\csc$ .
16. Pythagorean identities.
17. Example: Given  $\sin x = 4/5$  and  $P(x)$  is in quadrant II find  $\tan$ ,  $\cot$ ,  $\sec$ ,  $\csc$ .
18.  $\cot x = -\tan(x - \pi/2)$  and the graph of  $\cot$ .

19. Some relations between the graphs of  $f(x)$  and  $1/f(x)$ .
20. The graphs of  $\csc$  and  $\sec$ .
21. Addition formulae for  $\sin$ .
- Find  $\sin 105^\circ$ .
  - Find  $\sin \frac{13\pi}{12}$ .
  - Geometric interpretation of complex multiplication.
22. Inverse trigonometric functions.
- Special values.
  - $\cos(\sin^{-1} x) = \sqrt{1-x^2}$  and variants.
  - $\cos^{-1} x = \sin^{-1}(-x) + \pi/2$ .  
Example:  $\cos^{-1}(-1/2)$ .
23. Double-Angle formulae.
- $\sin 2x = 2 \sin x \cos x$ .
  - $\cos 2x = \cos^2 x - \sin^2 x$ .
  - $\cos 2x = 2 \cos^2 x - 1$ .
  - $\cos 2x - 1 = -2 \sin^2 x$ .
  - Prove  $\cot x - \tan x = 2 \cot 2x$ .
  - Express  $\cos 3x$  in terms of  $\cos x$ .
  - Examples:
    - Find  $\cos(2 \cos^{-1} 4/5)$ .
    - Find  $\tan(\tan^{-1} 2 + \tan^{-1} 3)$ .
24. Half-Angle Formulae
- $\sin^2 \frac{x}{2} = \frac{1 - \cos x}{2}$ .
  - $\cos^2 \frac{x}{2} = \frac{1 + \cos x}{2}$ .
25. Formulas involving the tangent.
- $\tan(s+t) = \frac{\tan s + \tan t}{1 - \tan s \tan t}$ .
  - $\operatorname{sgn} \tan \frac{\theta}{2} = \operatorname{sgn} \sin \theta$ .
  - $\tan \frac{\theta}{2} = \frac{1 - \cos \theta}{\sin \theta}$ .
- $\tan \frac{\theta}{2} = \frac{\sin \theta}{1 + \cos \theta}$ .
- (e) Geometric interpretations of half and double angle formulae for  $\tan$ .
- (f) Example. Find  $\tan \frac{7\pi}{8}$ .
26. Product to Sum Formulae
- $\sin s \sin t = \frac{1}{2}[\cos(s-t) - \cos(s+t)]$ .
  - $\cos s \cos t = \frac{1}{2}[\cos(s-t) + \cos(s+t)]$ .
  - $\sin s \cos t = \frac{1}{2}[\sin(s-t) + \sin(s+t)]$ .
27. Example. Express  $2 \cos t \cos 2t$  as a sum.
28. Sum to product formulae.
29. Trigonometrical Equations:
- $\sin x = -1/2$ .
  - $\sin 3x = -1/2$ .
  - $\cos x = 1$ .
  - $4 \cos^2 x = 1$ .
  - $\cos 2t = \cos t$ .
  - $\sin 2x + \cos 2x = 1$ .
  - $4 \sin t = 3 \csc t$ .
  - $2 \cos^2 \theta + 5 \cos \theta - 3 = 0$ .
30. Identities
- $|\sin x + \cos x| = \sqrt{1 + \sin 2x}$ .
  - $\tan \frac{\theta}{2} = \frac{\sin \theta}{1 + \cos \theta}$ .
31. Triangles.
- Polar coordinates and polygonal paths.
  - Triangular paths and the sine rule.
  - Right triangles.
  - Solution of triangles  $SAA$  and  $SSA$  by the sine rule.
  - Solution of triangles  $SSS$  and  $SAS$  by the cosine rule.
  - Area of a triangle.