Math 4995 Prof. Pennance –Basic Combinatorial Results for Finite Sets

1. Pigeon-hole Principle
   (a) If $\phi : \mathbb{N}_m \to \mathbb{N}_n$ is injective then $m \leq n$
   (b) If $\phi : A \to B$ is injective, then $|A| \leq |B|$
   (c) If $|A| > |B|$, there is no injection $\phi : A \to B$
   (d) If $\phi : A \to B$ is a surjection, then $|A| \geq |B|$
   (e) If $a \in A$, then $|A| \setminus a| = |A| - 1$
2. Inclusion-Exclusion
   (a) If $|A \cap B| = \emptyset$, then
   $$|A \bigcup B| = |A| + |B|$$
   (b) $A \cup B| = |A| + |B| - |A \cap B|$
3. Product Formulae
   (a) $|A \times B| = |A||B|$
   (b) $|A^n| = |A|^n$
   (c) $|A^{\{1,2,\ldots n\}}| = |A|^n$
   (d) $\mathrm{TO}(A) = |A|^n$
4. Counting Subsets
   (a) $|2^A| = 2^{|A|}$
   (b) If $|A| = |B|$, then $|\binom{A}{k}| = |\binom{B}{k}|$
   (c) $|\binom{A}{k}| = \binom{|A|}{k}$
   (d) $(x+y)^n = \sum_{i=0}^{n} \binom{n}{i} x^i y^{n-i}$
   (e) $\mathrm{Bip}(A) = 2^{|A|-1}$
5. Counting Functions
   (a) If $|A| = |B| = n$, then $|\mathrm{Bij}(A, B)| = n!$
   (b) $|A!| = |A|!$
   (c) $|A^B| = |A|^{|B|}$
   (d) $P(A, k) = P(|A|, k)$