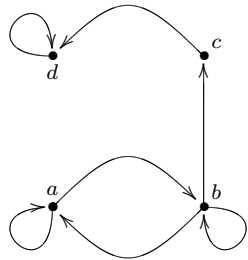


1. A *directed graph* is a pair (V, E) where V is a non empty set of elements called *vertices* and E is a binary relation on V , (i.e. E is a subset of $V \times V$). The elements of E are called *directed edges*.
2. Example. Let $V = \{a, b, c, d\}$ and $E = \{(a, a), (b, b), (d, d), (a, b), (b, a), (b, c), (c, d)\}$. Then $G = (V, E)$ is a directed graph. G is depicted below.



3. Notation. Let $G = (V, E)$ be a directed graph and $x, y \in V$. We write xEy or $x \rightarrow y$ to indicate that $(x, y) \in E$.
4. A binary relation is called:
 - (a) *reflexive* if aEa for all $a \in V$.
 - (b) *irreflexive* if aEa for no $a \in V$.
 - (c) *symmetric* if aEb implies bEa , for all $a, b \in V$.
 - (d) *antisymmetric* if aEb and bEa imply $a = b$, for all $a, b \in V$.
 - (e) *transitive* if aEb and bEc imply aEc for all $a, b, c \in V$.
 - (f) a *pre-order* if it is reflexive and transitive.
 - (g) an *order* if it is reflexive, transitive and antisymmetric.
- (h) an *equivalence relation* if reflexive, transitive and symmetric.
5. In a preorder we say that:
 - (a) a is *less than* b and write $a < b$ if aEb and not bEa .
 - (b) a is *greater than* b and write $a > b$ if bEa and not aEb .
 - (c) a is *incompatible* with b and write $a \parallel b$ if neither aEb nor bEa .
 - (d) a is *equivalent to* b and write $a \sim b$ if both aEb and bEa .
6. Let a, b be a pair of vertices in an pre-order, then, clearly, exactly one of the following is true:
 - (a) $a < b$,
 - (b) $b < a$,
 - (c) $a \parallel b$,
 - (d) $a \sim b$
7. In an equivalence neither $a < b$ nor $b < a$ are possible by symmetry. Hence, in an equivalence relation, $a \sim b$ is synonymous with aEb .
In an order relation we cannot have $a \sim b$ unless $a = b$. Hence, in an order relation, $a \leq b$ is synonymous with aEb .
8. This justifies the following alternative notations for $(a, b) \in E$.
 - (a) $(a, b) \in E$ (used for arbitrary digraphs)
 - (b) $a \lesssim b$ (used in preorders)
 - (c) $a \sim b$ (used in equivalence relations)
 - (d) $a \leq b$ (used in orders)