Math 4995 Prof. Pennance – Exercises on Functions

1. Find a set B such that the triple $F = (\mathbb{R}, B, \{(x, x^2 - 2x) : x \in \mathbb{R}\})$ is a surjective function. Find a right inverse of F.

2. Find a set $A \subseteq \mathbb{R}$ such that the triple $(A, \mathbb{R}, \{(x, x^2 - x - 6) : x \in A\})$ is a injective function. Find a left inverse of F.

3. Find sets $A \subseteq \mathbb{R}$ and $B \subseteq \mathbb{R}$ such that the triple $F = (A, B, \{(x, 4\sin 2x) : x \in A\})$ is a bijective function. Find the inverse function of F and draw its graph.

4. Find a set $A \subseteq \mathbb{R}$ such that the triple $(A, \mathbb{R}, \{(x, 5) : x \in A\})$ is a injective function. Find a left inverse of F.

5. Let S be the set of circles in the plane and let $f : S \to \mathbb{R}$ be defined by $f(S) =$ the area of S. Is f injective?, surjective?, bijective?.

   Now let T be the set of circles in the plane whose center is the origin and define $g : T \to \mathbb{R}^+$ by $g(T) =$ the length of the circumference of T. Is g injective? surjective? bijective?. Justify your answers.

6. Let $\mathbb{Z}$ be the set of integers and $\mathbb{Q}$ the set of rational numbers.Find all triples a, b, c of rational numbers such that $(\mathbb{Z}, \mathbb{Z}, \{(x, ax^2 + bx + c) : x \in \mathbb{Z}\})$ is a function.

7. Let $\mathbb{Z}$ be the set of integers and $\mathbb{Q}$ the set of rational numbers. Let $G = \{(m/n, m) : m, n \in \mathbb{Z}, n \neq 0\}$. Is the triple $(\mathbb{Q}, \mathbb{Z}, G)$ a function? (Justify your answer)

8. Let $G = \{(\sqrt{x}, \sin x) : x \in [0, 4]\}$. Is the triple $([0, 2], \mathbb{R}, G)$ a function? Prove your answer.

9. Let $G = \{(\sin x, x^2) : x \in [0, \pi]\}$. Show that the triple $([0, 1], \mathbb{R}, G)$ is not a function.

10. Let

    $$G = \{(\sin x, (x - \pi/2)^2) : x \in [0, \pi]\}.$$

    Show that the triple $([0, 1], \mathbb{R}, G)\}$ is a function.

11. Let $f : A \to B$ and $g : A \to C$ and $G = \{(f(t), g(t)) : t \in A\}$. Find sufficient conditions for the triple (B, C, G) to be a function.

12. Let $f : \mathbb{Z} \times (\mathbb{Z} \setminus 0) \to \mathbb{Q}$ be the function with values given by $f(m, n) = m/n$. Let $g : \mathbb{Z} \times (\mathbb{Z} \setminus 0) \to \mathbb{Z}$ be the function with values given by $g(m, n) = m$. Let $G = \{(f(m, n), g(m, n)) : (m, n) \in \mathbb{Z} \times (\mathbb{Z} \setminus 0)\}$. Show that the triple $(\mathbb{Q}, \mathbb{Z}, G)$ is not a function.

13. Let $f : A \to B$ and $g : B \to C$. Show that $g \circ f$ surjective implies that g surjective.

14. If $f : A \to B$ and $g : B \to A$ satisfy $f \circ g = id_B$ show that f is surjective and g is injective.

15. Let $f : A \to B$ and $g : B \to C$. Show that $g \circ f$ surjective implies that g surjective.

16. Let $f : A \to B$ and $g : B \to C$. Show that $g \circ f$ surjective and g injective imply that f is surjective.