

Trigonometric functions

- Find the exact values of:
 - $\sin(61\pi/6)$
 - $\sin(62\pi/6)$
 - $\sin(63\pi/6)$
 - $\sin(64\pi/6)$
 - $\sin(65\pi/6)$
 - $\sin(67\pi/6)$
 - $\sin(68\pi/6)$
 - $\sin(69\pi/6)$
- If $\sin \alpha = 4/5$, $\sin \beta = 3/5$ and $0 < \alpha, \beta < \pi/2$, find the exact value of $\cos(\alpha - \beta)$.
- If $P(\theta)$ lies on the line with equation $y = mx$ and $\pi/2 < \theta < \pi$. Find $\sin \theta$, $\cos \theta$, and $\tan \theta$ in terms of m .
- Find $\sin \phi$ given that $\cos \phi = -\frac{1}{4}$ and ϕ is (the measure of) a third quadrant angle.
- Simplify $(7 \sin x - \cos x)^2 + (\sin x + 7 \cos x)^2$
- Express as linear combinations of $\sin x$ and $\cos x$:
 - $\sin\left(x + \frac{7\pi}{2}\right)$
 - $\cos(42\pi - x)$
 - $\sin(53\pi + x)$
 - $\cos\left(x - \frac{3\pi}{2}\right)$
- Express $\sin\left(x - \frac{3\pi}{2}\right) + \cos(43\pi - x)$ as linear combinations of $\sin x$ and $\cos x$.

Graphs

- Sketch the graphs of the following functions
 - $t \mapsto \sin(2t)$
 - $t \mapsto \sin\left(\frac{t}{2}\right)$
 - $t \mapsto 2 \sin t$
 - $t \mapsto 2|\sin t|$
 - $\xi \mapsto \sin 2|\xi|$
 - $\phi \mapsto \tan |\phi|$
 - $\Phi \mapsto |\tan \Phi|$
 - $\mu \mapsto \cos |\mu|$
 - $\gamma \mapsto |\cos \gamma|$

Sum formulae

- Find 4 distinct pairs x, y of real numbers for which

$$\sin(x + y) = \sin x + \sin y$$
- Find real numbers A, ω and δ such that for all $t \in \mathbb{R}$

$$\sin t + \cos t = A \sin(\omega t + \delta)$$
- If $\tan A = 1/4$ and $\cos B = 1/3$, where A, B are acute angles, find $\sin(A - B)$.
- If $\tan A = 3/4$ and $\cos B = 12/13$, and A, B are acute angles, find the value of $\cos(A + B)$.
- Write $\sin 3x$ in terms of $\sin x$.

Half angle formulae

- Find the exact values of:
 - $\sin \frac{\pi}{12}$
 - $\cos \frac{\pi}{12}$
- Prove $\forall \theta \in \mathbb{R}, \operatorname{sgn}(\tan \frac{\theta}{2}) = \operatorname{sgn}(\sin \theta)$
- Prove $\tan \frac{\theta}{2} = \frac{1 - \cos \theta}{\sin \theta}$
- Find a geometric interpretation of the identity in (3).

Identities

- Prove:
 - $\sqrt{1 + \sin 2x} = |\sin x + \cos x|$
 - $(\sin x)(\tan x) = \sec x - \cos x$.
 - $(a \cos x + b \sin x)^2 + (b \cos x - a \sin x)^2 = a^2 + b^2$.

Inverse functions

1. If the expression represents a quantity which exists find its exact value:

- (a) $\sin^{-1}(\sin 4)$
- (b) $\sin^{-1}(\cos(0))$
- (c) $\cos^{-1}(\cos 4)$
- (d) $\sin^{-1}(\sin \pi)$
- (e) $\sin(\sin^{-1} 4\pi)$
- (f) $\sin(\sin^{-1} \frac{3\pi}{5})$
- (g) $\cos(\sin^{-1} \frac{1}{5})$
- (h) $\sin(\tan^{-1}(-\frac{1}{3}))$
- (i) $\sin(\cos^{-1}(-\frac{1}{3}))$
- (j) $\sin(\cos^{-1} \frac{1}{3})$

Product formulae

1. Write $\cos 2\phi \cos 3\phi$ as a sum.

Trigonometric equations

1. Solve the following:

- (a) $\csc x = \frac{1}{2}$

- (b) $2 \cos \alpha - 3 = 0$
- (c) $\sin 2\beta = \cos \beta$
- (d) $\sin \gamma = \sin(-\gamma)$
- (e) $\cos(-t) = -\cos t$
- (f) $3 \sin^2 \mu - 4 \cos \mu + 1 = 0$
- (g) $(\log_3 \cos \Theta)^2 - \log_3 \cos^3 \Theta - 4 = 0$
- (h) $3 \cos \Phi = 2 \sin^2 \Phi$
- (i) $(\log_2 \sin \nu)^2 + \log_2 \sin^4 \nu + 4 = 0$
- (j) $\sin 2x + \cos 2x = 1$

2. Find the number of zeros of the function $f(x) = \sin 10x$ in the interval $[0; \pi]$
3. Find the number of solutions in the interval $[0; 10\pi]$:

- (a) $\cos \frac{x}{3} = -1$
- (b) $\cos \frac{x}{3} = -1/2$

4. Find the smallest positive zero of the function $f(x) = \sin(4x - 3)$.
5. Do there exist numbers x, y such that $\tan(x + y) = \tan x + \tan y$? Investigate.