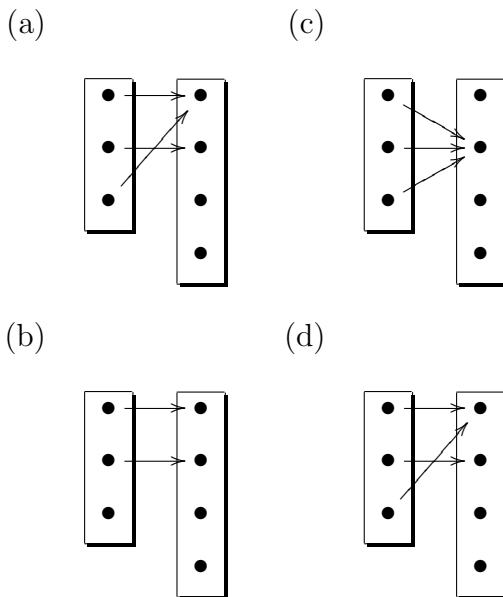


Math 3018 Prof. Pennance – Injective and surjective functions

1. A function f is called *injective* if in the corresponding bipartite graph the degree of every vertex in the codomain has degree at most 1 (i.e. degree ≤ 1).
2. The following are equivalent:
 - (a) f is *injective*
 - (b) For any elements $a, a' \in \text{dom } f$, if $a \neq a'$ then $f(a) \neq f(a')$.
 - (c) For any elements $a, a' \in \text{dom } f$, if $f(a) = f(a')$ then $a = a'$.
3. Examples. Consider the relations represented by the bipartite graphs in the drawing below. Only (d) depicts an injective function. The relation depicted by drawing (b) is not a function so in particular is **not** an injective function even though the degree of every vertex in the codomain has degree at most 1.



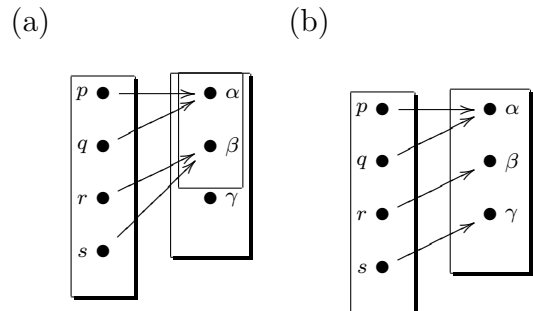
4. The *image* or *range* of a function f (denoted $\text{im}f$) is the set of vertices in the

codomain whose degree (in the corresponding bipartite graph) is at least 1. Since $f(x)$ is the unique vertex adjacent to x in this graph, it follows that

$$\text{im}f = \{f(x) : x \in \text{dom}f\}$$

5. A function f is called *surjective* if its codomain and image coincide. The following statements are equivalent.
 - (a) f is *surjective*
 - (b) For every element b in the codomain of f , there exists an element a in the domain of f such that $f(a) = b$.
 - (c) In the corresponding bipartite graph, every vertex in the codomain has degree at least 1.

6. Examples. Consider the functions whose bipartite graphs are drawn below:



Both functions have codomain $\{\alpha, \beta, \gamma\}$. The image of the function in diagram (a) namely $\{\alpha, \beta\}$ is not equal to the codomain so this function is not surjective. Since the image and codomain of the function depicted in diagram (b) coincide this function is surjective.