Concerning the Measurability of a Global Mean Surface Temperature

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Models of global climate change typically assume the existence of a global mean surface temperature. Difficulties in estimating this parameter, as evidenced by discrepancies between different studies of the earth's surface temperature record, have generated questions concerning its reliability as a climate change metric. This paper examines theoretical explanations for these discrepancies. In particular, it is shown that no finite set of temperature readings can ever reliably approximate the mean value of an arbitrary but **unknown** temperature distribution over the surface of a sphere.

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I. INTRODUCTION

Simple theoretical models of the earth's climate based on the thermodynamics of homogeneous fluid bodies, or on statistical mechanics, predict that certain gases such as CO_2 and methane can trap long wavelength radiant energy leading to temperature increases. Interesting examples of such models are given in [1] together with an excellent general introduction to the science of global warming.

Assertions which are correctly derived from simple mathematical models have the status of theorems insofar as they are rigorously true in any system in which the model assumptions are satisfied. Problems arise, however, when attempting to apply such models to a multi-factorial, inhomogeneous, non-equilibrium, thermo-mechanical and chemical system, such as the earth's climate, whose complexity far exceeds that of any feasible model. Even when model assumptions are satisfied, there may exist variables which are overlooked. Simulations of climate based upon such models may also be computationally intractable and/or sensitive to initial conditions, making realistic climate predictions impossible. When models are sensitive to initial conditions, computer simulations serve more as measures of model sensitivity than as predictors of climate.

One of the main parameters used in climate models is the global mean surface temperature and various temperature data sets [3, 4] have been collected and used to estimate global averages [5, 10]. Unfortunately, the earth's surface temperature record has a number of well-documented flaws and much controversy exists concerning the measurement of this parameter. For a general overview of this and related climate issues see [7–9].

The purpose of this article is to examine some theoretical and philosophical issues concerning the definition and measurement of the parameter global mean surface temperature. In Section II it is shown that the property of global warming as determined by a global mean surface temperature is scale dependent and that the same set of data can actually imply both global warming and global cooling depending on how the temperature scale is defined. Section III describes how measurement procedures based on the central limit theorem can permit a distinction to be made between discrepancies due to a poor model and those caused by experimental error. It is shown that measurements of global mean surface temperature are not of this type and, consequently, that bias in the measurement process may be intrinsic and not amenable to correction by, say, an increase in the number of data points or by improved statistical processing. Finally, Section IV presents explicit examples of temperature distributions over a sphere which have the property that, with probability 1, any finite or countably infinite set of temperature measurements will estimate a global mean temperature differing from the true mean by an arbitrary large predetermined amount.

The issues raised herein shed some light on the current discrepancies between different estimates of global mean surface temperature. They also highlight the need for a careful theoretical evaluation of this concept and its reliability as a climate change metric.

II. THE RELATIVITY OF TEMPERATURE

In a well known conversation with Heisenberg cited in [2], Albert Einstein stated that

"... On principle it is quite wrong to try founding a theory on observable magnitudes alone. In reality the very opposite happens. It is theory which decides what we can observe."

This remark applies as much to thermodynamics as it does to the theory of relativity. To illustrate why, we start with a parable.

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A. A Fable of Global Warming

On a certain planet called X, the average temperature had been a steady $16°X$ *for as long as anyone could remember. During a spell of particularly X-clement weather, a climatologist from X took two temperature measurements and obtained the X-treme values of* $0°X$ *and* $36°X$ *for a mean of* $18°X$ *–a full* $2°X$ *above average. Fully cognizant of the fame and pecuniary advantages accruing to the discoverer of such an important finding, the scientist proceeded to write a grant proposal outlining a theory of X-kind induced global warming –of course, taking great pains to hint at possible serious implications for X-habitants if this phenomenon was not immediately remedied by lifestyle changes which might entail drastic X-onomic changes.*

News of this "discovery" was published in the local newspaper (aptly named The X-PRESS) and transmitted (by X-ray) to neighboring regions of the galaxy. Before (relatively) long, a scientist from the distant planet Zeldon arrived on X to investigate this phenomenon. Being unfamiliar with degrees X, he immediately set about converting all the temperature readings to degrees Z, the unit of temperature on Zeldon. Since the two scales were related by the square root function this was not unduly difficult –the data values of $0°X$ *and* $36°X$ *obtained by the X-habitant were quickly converted to* $0°Z$ *and* $6°Z$ *giving an average of* $3°Z$*. Noting that the previous average temperature* $16°X$ *converted to* $4°Z$*, the Zeldonian announced that there existed absolutly no evidence of global warming but, rather, that the empirical data actually indicated global cooling.*

A diplomatic incident ensued when the Zeldonian imputed that scientists from X were incompetent and that their thermometric scales were not "intelligently designed". In a heated X-change, the climatologist from X told the Zeldonian "Our thermometric fluid has been very thoroughly studied. It is so well designed that the length of a column of this liquid enclosed in a narrow glass tube varies linearly with our temperature. It is such a versatile substance that it is even used as a dental amalgam on our planet!" "No wonder you X-ians need root canals. No competent scientist could possibly consider using a thermometric fluid which yields the square of the temperature in degrees Z. What do you call this crazy substance anyway?", queried the Zeldonian. "It is named after Mercury, a nearby planet in our solar system", replied the X-habitant. Unmoved by all arguments, the Zeldonian proceeded to write a grant proposal to study the phenomenon of global cooling. Meanwhile, on a distant planet, in a distant galaxy, a different scientist using a different temperature scale failed to detect any change in the average temperature of X -and lost his tenure for not publishing.

B. On the Transformation of Temperature Scales

The seeming paradox in the above fable arises as a result of the failure of nonlinear changes of temperature scale to preserve average temperatures. Common sense dictates that there is not much physical meaning in averaging the temperatures of the sun and a beaker of ice water. Unfortunately, in this age of readily available technology, such computations are far too common. The truth is, as Einstein was very well aware, that the axioms of a given physical theory can, and frequently do, impose restrictions on the class of functions that can be used to make changes of scale. This, in turn, imposes limits on the statistical quantities that can be "meaningfully" computed. For example, the requirement that the total length of a concatenation of two objects be the sum of their lengths, together with a continuity assumption, leads to the restriction that changes of length scale **must** be linear (see Appendix A). Thus, to change yards to feet we use the transformation $L' = 3L$, a linear operation. Since linear functions preserve the arithmetic mean, under a change of length scale from, say, yards to feet, an average length in yards will transform correctly to an average length in feet. Thus, the concept of average length is scale independent and statistically meaningful.

Temperature, on the other hand, does not behave in the same way as length with respect to concatenations. For example, two beakers of water, each at temperature $10°C$ do not combine to produce a concatenation with temperature $20°C$ and so the physical condition forcing linearity in the case of changes of length scale does not apply. Consequently (see Appendix A), changes of temperature scale are not limited to linear functions. Unfortunately, there is a fairly widespread misconception that changes of temperature scale must be affine, of the form $T' = aT + b$ where $a > 0$. This idea probably arises, by false analogy, from the fact that commonly used changes in scale, such as that from Fahrenheit to Celsius, are of this form. In reality, the only constraint, in classical thermodynamics, governing changes of temperature scale is the assumption that the set of all hotnesses of a body comprise what mathematicians call a one-dimensional C^1 manifold with an intrinsic ordering [12]. It is well known (see Appendix A) that this axiom is compatible with any change of temperature scale which is continuous and order preserving, such as the square root function used in the fable. Since an arbitrary order preserving change of scale does not in general preserve the mean, it becomes possible, depending on the scale used, to interpret the same set of data as either global warming or global cooling.

Thus we see that the physical meaningfulness of a quantity depends crucially on the allowed changes of frame and scale. If a person were so lucky as to double his money, it would have roughly the same consequences in any currency; but the effects of a Celsius doubling of temperature would in general be different from a doubling in degrees Fahrenheit. Moreover, if non-affine changes of temperature scale are permitted, it makes no sense to say, for example, that the temperature difference between 0 and 10 degrees Celsius is the same that between 10 and 20 degrees Celsius. In a nonlinear world, temperature differences as well as average temperatures are scale dependent quantities.

The fact that, in practice, we limit ourselves to affine changes of temperature merely masks this issue, giving a false sense of security that the averaging of temperatures, or the comparing of temperature differences, are actually meaningful. However, the underlying philosophical concerns remain, and, insofar as they prohibit nonlinear operations in the processing of temperature records (see for example [10]), are relevant to the study of climate change.

III. GLOBAL MEAN SURFACE TEMPERATURE: A MEASUREMENT PROBLEM

Agreement concerning basic definitions, models, and measurement procedures should be an objective of good science. Consider, for example, the concept of length in a Galilean spacetime. If a sequence of independent, identically distributed measurements $X_1, X_2, \ldots X_n$ is made of the length μ of a rigid rod, then, by the central limit theorem, the sampling distribution of the mean $\bar{X}$ of the sequence converges to a normal distribution with mean equal to the mean of the sample distributions. In the absence of bias, the mean of the sample distribution will be the exact distance μ. Moreover, the variance of the data can be compared to that expected from knowledge of experimental errors inherent in the details of the measurement process. Unexpected discrepancies can then be evaluated and possible explanations studied.

Application of this measurement procedure to, for example, the temperature measurement of a homogeneous fluid body in a constant state would quite likely raise few concerns. On the other hand, the surface of the globe is not a homogeneous fluid body in a constant state. If a number of different observers each using different sets of surface points make estimates of the global mean surface temperature, then the sequence of values so obtained does **not** represent a list of independent, identically distributed random variables and the hypotheses of the central limit theorem do not hold. Rather, this situation is like unto a set of observers, ignorant of the precepts of special relativity, travelling at different, but high, velocities relative to each other, and attempting to ascribe a length to a rigid rod. Since the model hypothesis of a Galilean spacetime is false, the very concept of length (other than rest length) of a rod is problematical, although our observers are not aware of this. In this situation, differences in length obtained by different observers cannot be attributed merely to technical difficulties connected with the measuring process but rather to the incorrect model assumption of a Galilean spacetime. Due to the relativity of simultaneity, different observers are, in reality, measuring the separations between different pairs of spacetime events. Discrepancies obtained by such observers cannot in this case be ascribed to experimental error which can in principle be overcome by sufficient ingenuity; rather, their explanation requires a fundamental change of philosophy and the recognition that each is measuring a different parameter. In this case, knowledge of the experimental errors involved and of the discrepancies between observers would hopefully lead to a change in the model hypothesis and the discovery of special relativity. This, in turn, would enable the observers to transform their measurements to the rest frame of the rod, thereby yielding consistent estimates of the true rest length.

Unfortunately, the measurement of global mean surface temperature turns out to be far more complex than the measurement of a length and the discrepancy due to different observers measuring different distributions cannot be so easily transformed away. Some authors, (see for example [6]), attribute statistical bias in global mean temperature estimates to an uneven distribution of weather stations. This attribution implicitly assumes that such errors are amenable to correction by more uniform sampling or, perhaps, by better statistical processing. However, as will be shown in Section IV, the real problem is more fundamental and is essentially due to the existence of temperature distributions defined on the sphere with the property that with probability 1 **any** sample will yield a global mean surface temperature with as large a pre-prescribed degree of bias as desired. Such measure-theoretic bias is not amenable to correction by increased sample size or improved sample distribution or by a suitable smoothing of the data. Worse still, many features of the earth's actual temperature distribution suggest that it shares this type of pathology.

IV. GLOBAL MEAN SURFACE TEMPERATURE: AN ESTIMATION PROBLEM

In this section it is shown, by counterexample, that no finite set of temperature readings can reliably approximate the mean value of an arbitrary but **unknown** temperature distribution over the surface of a sphere. To quantify this assertion, it suffices to show that there exist infinitely many temperature distributions over the sphere, each one possessing a well-defined mean, and with the property that any countable random sample will, with probability 1, have a mean which differs from the true mean by a pre-specified error Δ. But it is straightforward to exhibit such distributions. Just let the temperature over the surface of the sphere be an arbitrary constant say c except at a single point P at which the temperature is a δ functional of weight normalized so that the average temperature over the sphere is exactly equal to $\Delta + c$. Then any uniform random sample of points on the sphere would, with probability 1, fail to contain the point P and so yield a mean temperature of c. Thus, the sample mean would differ from the true mean by exactly the amount Δ. Infinitely many variants of this construction can easily be conceived and, moreover, recourse to the theory of distributions is inessential in the construction. We need only replace the requirement that the probability be exactly 1 by $1 - \epsilon$ where $0 < \epsilon < 1$ is as small as we want and permit the temperature spike to extend over a suitable area depending on ϵ. It cannot be argued that such distributions are atypical of our earth. Temperature spikes do occur in the form of volcanoes. Less sharp temperature jumps can be found, for example, at icebergs floating in warm water and at shorelines which are ever shifting and changing. It is clear that refinements of this basic argument can be made for almost any realistic global temperature distribution.

In reality, when temperature is measured, there exists the eminently practical problem of deciding how much weight to allocate to sample points which might be so unlucky as to fall in such hostile and inconvenient places as on an iceberg in the middle of the ocean or at the center of a vocano. Two icebergs or two volcanoes of the same "surface area" may have considerably different volumes and, hence, correspondingly different thermodynamic contributions. The human decisions necessary in such circumstances, indeed, inevitably lead to a degree of arbitrariness. This arbitrariness is compounded by the additional difficulty, not dealt with here, that when time averages are computed, the values obtained depend on the time interval selected by the observer to compute them and hence are subject to human bias.

The above constructions suggest that bias due to the sampling process in the measurement of global mean surface temperature is intrinsic and cannot be overcome by, say, increases in sample size and quality. In reality, the bias is measure-theoretic in origin. A finite sample of points has surface measure zero and cannot, in general, reliably estimate an unknown surface distribution. A surface has volume measure zero and surface measurements cannot reliably estimate an unknown volume distribution.

V. CONCLUSION

Much effort has been expended in recent years in attempts to measure and model changes in the global climate and, in particular, the global mean surface temperature. Since models suggest that human activities may, in fact, play a significant role in climate change, the issue has become one of political as well as scientific importance and discrepancies in empirical data and disparities between models have fueled much controversy. This article has outlined some possible explanations for these discrepancies. In particular, it has highlighted theoretical difficulties concerning the existence, meaning and measurement of a global mean surface temperature. The analysis suggests that current discrepancies in the temperature record cannot be eliminated by the mere introduction of more weather stations and better data processing but, rather, that they are more fundamental in nature. It underscores the necessity for a careful mathematical and philosophical scrutiny of the concept of global mean surface temperature and its use as a climate change metric.

Because of the current heated controversy concerning the link between climate change and human activities and the concomitant possibility of misinterpretation, it is worth stating what this article does not claim. It in no way disputes the essential correctness of heat trapping models. As stated in the Introduction, to the degree to which model predictions are mathematically rigorous, they are theorems and, hence, true in any system in which their hypotheses hold. In the real world, which is much more complex, model features may or may not reflect aspects of real climate processes. That anthropogenic greenhouse gases are one factor in the complex multi-factorial system comprising our climate(s) is indisputable. It is clear that any change in atmospheric compositions will affect absorptivity and, in turn, the planetary climate. But in a system as sensitive to initial conditions as the earth's climate, almost all of the activities of man and beast will have such effects. What is of importance to ascertain is whether the effects of climate change are:

C1. Sufficiently pernicious to warrant radical human intervention.

C2. Susceptible to amelioration in a manner which is less pernicious than the problem.

This paper does suggest reasons to question recent widespread media assertions, often based on global mean temperature measurements, that the conjectures C1 and or C2 are no longer open. On the other hand, whatever the status of C1 and C2, actions such as sensible resource and energy conservation, pollution control, and the careful use of the earth's resources are good policy for many reasons, independently of any effects on the earth's climate.

APPENDIX A: STEVENS' CLASSIFICATION OF VARIABLES

Although well known, restrictions on changes of scale are often not given sufficient importance in science education. It is, indeed, ironic to find more discussion of this matter in Philosophy, Psychology, and Sociology texts than in Natural Science texts. In fact, it was a Harvard psychologist, S. S. Stevens [11], who produced a well known taxonomy of measurement scales based on the classes of transformation admissible under a change of scale. For the convenience of the reader, we include a brief outline of Stevens' taxonomy. In this classification a variable is called *ratio*, *interval*, *ordinal* or *nominal* according to the class of transformations permitted under a change of scale. The classes of admissible transformations are summarized in Table I.

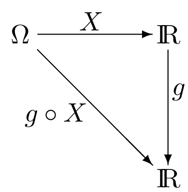
By means of examples from Physics we illustrate how each of these categories arises. We begin by briefly examining the transformation of random variables in general.

TABLE I: Classes of Variables

Variable Class	Admissible Scale Transformations	
Ratio	Linear	$g(t) = ct$, $c > 0$
Interval	Affine	$g(t) = ct + b$, $c > 0$
Ordinal	Order preserving	$t < t' \implies g(t) < g(t')$
Nominal	One to one	$t \neq t' \implies g(t) \neq g(t')$

1. Transformations of Random Variables

Let Ω be the set of all outcomes of a random experiment and let $X(\omega)$ be a random variable defined for each outcome $\omega \in \Omega$. If g is a real-valued function, then the composition $Y = g(X(\omega))$ defines a new random variable Y often referred to as the *transformed* random variable.



Such transformations have important mathematical and physical applications. Common statistical applications include standardization, grouping of data, changing the form of a distribution, and changes of frame and scale. For example, if X is a random variable with mean μ and variance σ, then $g(x) = (x - \mu)/\sigma$ transforms X to a "standardized" random variable Z with mean 0 and variance 1. Geometrically, $Z(\omega)$ is just the signed distance between $X(\omega)$ and the mean μ measured in standard deviations. If $g(t) = (t-\mu)^2$, then $Y(\omega) = g(X(\omega))$ is a random variable whose mean EY is just the variance of X. If X is a continuous random variable and g a step function approximating the identity function, then the transformed variable Y will be a discrete variable with parameters and characteristics not too different from those of X, a fact useful in the grouping of experimental data and its depiction by, say, a histogram. In Physics, g can represent a Galilean or Lorentzian change of frame. What mainly concerns us here, however, are changes of scale.

2. Linear Changes of Scale

When the transformation of a random variable represents a change of scale of some quantity, then the axioms governing that quantity can impose restrictions on the functions g that may be used to transform the given variable. For example, if $X(B)$ represents the length in feet of a body B and $g(x) = 12x$, then $Y = g(X(B))$ is the length in inches. However, if $g(x) = \sqrt{x}$ then the variable $Y(B) = g(X(B))$ is not admissible as a measure of length. We now show that if the length of a concatenation $B \oplus B'$ of two bodies B and B' is equal to the sum of their lengths, i.e.,

$$X(B \oplus B') = X(B) + X(B) \tag{A1}$$

then the only admissible changes of scale for length are linear functions $Y = cX$ of positive slope c. In equation A1, the binary operation $\oplus$ of concatenation does not combine numbers but rather $A \oplus B$ represents the new body formed by the juxtaposition of the two objects A and B. Now, if $Y(B) = g(X(B))$ is a new measure of length (in the rest frame) of a body B, we require that in the new scale:

$$Y(B \oplus B') = Y(B) + Y(B').$$

That is,

$$g[X(B \oplus B')] = g[X(B)] + g[X(B')]. \tag{A2}$$

From A1 and A2 we have

$$g[X(B) + X(B')] = g[X(B)] + g[X(B')].$$

If this is to hold for bodies B and B' of arbitrary length, then the function g must satisfy:

$$g(x + y) = g(x) + g(y) \tag{A3}$$

for all $x, y \geq 0$. As is well known, the additivity condition A3 is not quite enough to ensure the linearity of g. However, if the lengths of a sequence of bodies tend to zero in one scale, then they tend to zero in any scale. This additional physical axiom leads us to impose upon g the additional condition that

$$\lim_{h \to 0^+} g(h) = 0 \tag{A4}$$

i.e., that g is continuous from the right at the origin. As is well known [13], this additional assumption is sufficient to prove that g must be linear $g(x) = cx$. Moreover, since length is non negative $c = g(1) > 0$. This proves that length is a ratio variable in Stevens' taxonomy.

Ratio variables arise whenever objects can be concatenated in such a way that equation A1 holds. Examples of such variables in Physics are time intervals, angle, mass, etc. However, variables such as position on a time scale and position on a length scale are not ratio variables since their admissible class of changes of scale is larger.

3. Affine Changes of Scale

A random variable is called *interval* if it admits only changes of scale $Y(\omega) = g(X(\omega))$ where g is affine:

$$g(t) = at + b, \quad a > 0. \tag{A5}$$

For example, if $X = X(P)$ is the coordinate of a point P on a length scale and X_0 is a fixed point on that scale, then $X - X_0$ is a length which we already know to be a ratio variable. It follows that if Y and Y_0 are the corresponding points after a change of scale, then $Y - Y_0 = a(X - X_0)$ where $a > 0$. Thus, $Y(P) = g(X(P))$ where g is affine of the form A5. This proves that position X on a length scale is an interval variable in Stevens' taxonomy. Physically, the change of scale $x \mapsto ax+b$ corresponds to a change of unit of length $x \mapsto ax$ followed by a change of origin $x \mapsto x + b$.

In general, if the distance of a point from the origin of some scale is a variable of ratio type, then the corresponding position of that object on the scale will be a variable of interval type. Thus, for example, the time of occurrence of an event is an interval variable whereas the time elapsed during some process is a ratio variable.

4. Ordinal Changes of Scale

In classical thermodynamics, the only restriction on a change of temperature scale is that it must preserve the order relation $\prec$ "hotter than" on the class of homogeneous fluid bodies. Since temperature X is a measure of degree of hotness, this implies that for any two such bodies B and B'

$$B \prec B' \Leftrightarrow X(B) < X(B').$$

Moreover, if it be demanded that this hold in any other temperature scale $Y(B) = g(X(B)$ we must also have

$$B \prec B' \Leftrightarrow Y(B) < Y(B')$$

and so

$$X(B) < X(B') \Leftrightarrow g(X(B)) < g(X(B')).$$

If this is to hold for fluid bodies B and B' of arbitrary temperature, the change of scale g must be a strictly increasing function. Since there are no other restrictions implied by the usual thermodynamic axioms, it follows that temperature is an ordinal random variable and not, as frequently claimed, interval. In particular, average temperature becomes a scale dependent object.

5. Arbitrary Changes of Scale

Finally, consider a quantity such as particle type

$$X = \begin{cases} 1 & \text{if type} = \Omega^- \\ 2 & \text{if type} = K\text{meson} \\ 3 & \text{if type} = \Sigma \text{ hyperon} \end{cases}$$

in which no particular order relation is specified on the set of particles. The variable X is now merely an indicator of category and the values of X are completely arbitrary. A change of scale such as $g(1) = 2$, $g(2) = 1$ which transposes the types Ω^- and K is perfectly admissible as indeed would any arbitrary bijection. Thus, in Stevens' taxonomy, X would be nominal.

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