MATH 3023 –Prof. Philip Pennance – Some truth tables

In the calulus of propositions (i.e. logic without quantifiers), if p, q are propositions, then so are the *compound* statements:

1. $p \vee q$ (*disjunction*).
2. $p \wedge q$ (*conjunction*).
3. $p \rightarrow q$ (*implication*).
4. $p \leftrightarrow q$ (*equivalence*).

The "truth" of these statements is determined by the truth of p and q according to the following tables known as *truth tables* in which 1 denotes *true* and 0 denotes *false*.

1. **Disjunction**

p	q	$p \vee q$
1	1	1
1	0	1
0	1	1
0	0	0

The disjunction $p \vee q$ is is false if both p and q are false, and true otherwise. If p is the statement "I teach mathematics" and q is the statement "I teach physics" then $p \vee q$ is the statement "Either I teach mathematics or I teach physics". Since disjunction of p and q is true when both p and q are true $\vee$ is sometimes called the *inclusive or*.

2. **Conjunction**

p	q	$p \wedge q$
1	1	1
1	0	0
0	1	0
0	0	0

The conjunction $p \wedge q$ is true if both p and q are true and false otherwise. If p is the the statement "4 is positive" and q is the statement "4 is less than 5", then $p \wedge q$ is "4 is positive and less than 5".

3. **Implication**

p	q	$p \rightarrow q$
1	1	1
1	0	0
0	1	1
0	0	1

The statement "p implies q" is *false* if p is true and q is false and true otherwise.

Important: The truth of a statement such as

"*If there is fire then there must be presence oxygen*"

or the falsehood of

"*presence of oxygen implies fire*"

can **NOT** be determined solely from the the above table. This point will be discussed in the lectures.

4. **Equivalence**

p	q	$p \leftrightarrow q$
1	1	1
1	0	0
0	1	0
0	0	1

The *equivalence* $p \leftrightarrow q$ is true if p and q have the same truth values and false otherwise. Linguistically $\leftrightarrow$ corresponds to the phrase " if and only if".