Math 3132 Prof. Pennance – Summary of Lecture XX - Units

1. Unit Prefixes:

Factor	Prefix	Symbol
10^9	giga	G
10^6	mega	M
10^3	kilo	k
10^2	hecto	h
1		
10^{-2}	centi	c
10^{-3}	milli	m
10^{-6}	micro	μ
10^{-9}	nano	n

2. Some conventions concerning units:

   (a) Measures should be expressed as an Arabic numeral followed by a blank space and then a unit. If the unit is a superscript the blank space should be omitted (e.g. 50°).

   (b) Only standard abbreviations should be used. (Warning: cc, mps, ppm etc. are **not** standard international units).

   (c) Italic type should be used for variables and quantity symbols, and Roman type for numbers and unit symbols.

   (d) Unit symbols should not be followed by a period except at the end of a sentence.

   (e) Ambiguities such as 2 × 3 cm should not be used but instead (2 × 3) cm in the case of length or 2 cm × 3 cm if area is intended.

   (f) A space or center dot (·) denotes the "multiplication" of units.

   (g) The plural and singular forms of unit symbols are the same.

   (h) Unit symbols and unit names should not mixed.

   (i) The symbol % should be used only to represent the number 0.01.

3. Examples:

Proper	Improper
s	sec
second	sec
cm^3	cc
l = 30 cm	l = 30 cms
s	sec
cm	*cm*
The rod is 35 cm long	The rod is 35 cm. long.
$4\ \mathrm{m}\cdot s^{-1}$	$4\ \mathrm{ms}^{-1}$
4 m/s	
2.0 μL/L	2 ppm
A body of mass 5 g	A mass of 5 g
$\mathrm{kg/m}^3$	$\mathrm{kilogram/m}^3$
25 kg	25kg
the current was 15 A	the current was 15 amperes
120°	120 °
10 cm × 10 cm	10 × 10 cm
$x = y(1 + 50\ \%)$	x exceeds y by 50 %

4. Conversion factors.

   (a) Let x and y be units of measure. The number of x in one y is called the *conversion factor* from y to x and is denoted $\left[\frac{\mathrm{x}}{\mathrm{y}}\right]$. Warning: Be careful of the order.

   (b) To convert units from y to x multiply by $\left[\frac{\mathrm{x}}{\mathrm{y}}\right]$.

   (c) Example. Convert 50 m to cm.

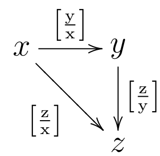
   Solution. Recall 100 cm = 1 m. Hence, by definition, $\left[\frac{\mathrm{cm}}{\mathrm{m}}\right] = 100$ is the conversion factor from m to cm. Therefore,

$$\begin{aligned} 50\ \mathrm{m} &= \left(50 \times \left[\frac{\mathrm{cm}}{\mathrm{m}}\right]\right)\ \mathrm{cm} \\ &= (50 \times 100)\ \mathrm{cm} \\ &= 5000\ \mathrm{cm}. \end{aligned}$$

5. Properties of conversion factors.

   (a)
$$\left[\frac{\mathrm{x}}{\mathrm{x}}\right] = 1$$

   (b)
$$\left[\frac{\mathrm{x}}{\mathrm{y}}\right] \cdot \left[\frac{\mathrm{y}}{\mathrm{z}}\right] = \left[\frac{\mathrm{x}}{\mathrm{z}}\right]$$



   (c)
$$\left[\frac{\mathrm{x}}{\mathrm{y}}\right] \cdot \left[\frac{\mathrm{y}}{\mathrm{x}}\right] = 1$$

   (which follows immediately from (a) and (b))

   (d)
$$\left[\frac{\mathrm{x}}{\mathrm{y}}\right] = \frac{1}{\left[\frac{\mathrm{y}}{\mathrm{x}}\right]}$$

   (rewrite of (c)).

6. Find the conversion factor $\left[\frac{\mathrm{s}}{\mathrm{hr}}\right]$ from hr (hours) to s (seconds).

   Solution: $\left[\frac{\mathrm{s}}{\mathrm{hr}}\right] = \left[\frac{\mathrm{min}}{\mathrm{hr}}\right] \cdot \left[\frac{\mathrm{s}}{\mathrm{min}}\right] = 60 \times 60 = 3600$.

7. Example. Given that $\left[\frac{\mathrm{ft}}{\mathrm{ml}}\right] = 5280$, convert 60 ml/hour (miles per hour) to ft/s (feet per second). Solution 1:

$$\begin{aligned}
\text{speed in ft/s} &= \frac{\text{distance in ft}}{\text{time in s}} \\
&= \frac{\left[\frac{\mathrm{ft}}{\mathrm{ml}}\right] \times \text{ distance in ml}}{\left[\frac{\mathrm{s}}{\mathrm{hr}}\right] \times \text{time in hr}} \\
&= \frac{\left[\frac{\mathrm{ft}}{\mathrm{ml}}\right]}{\left[\frac{\mathrm{s}}{\mathrm{hr}}\right]} \cdot \text{speed in ml/hr}
\end{aligned}$$

   Thus the conversion factor from miles per hour to feet per second is $\left[\frac{\mathrm{ft/s}}{\mathrm{ml/hr}}\right] = \frac{\left[\frac{\mathrm{ft}}{\mathrm{ml}}\right]}{\left[\frac{\mathrm{s}}{\mathrm{hr}}\right]} = \frac{5280}{3600} = \frac{22}{15}$ and so $60\,\mathrm{m/hr} = 60 \times \frac{22}{15}\,\mathrm{ft/s} = 88\,\mathrm{ft/s}$.

   Solution 2:

$$\begin{aligned}
60 \text{ ml/hr} &= (60 \times 5280) \text{ ft/hr} \\
&= \left(60 \times \frac{5280}{3600}\right) \text{ ft/s} \\
&= 88 \text{ ft/s.}
\end{aligned}$$

   Solution 3: Notice that the relation

$$\left[\frac{\mathrm{ft/s}}{\mathrm{ml/hr}}\right] = \left[\frac{\mathrm{ft}}{\mathrm{ml}}\right] \cdot \left[\frac{\mathrm{hr}}{\mathrm{s}}\right]$$

   justifies the rather informal "cancellation" of units seen in many chemistry texts.

$$\begin{aligned}
\frac{60 \text{ ml}}{1 \text{ hr}} &= \frac{60 \cancel{\text{ml}}}{1 \cancel{\text{hr}}} \cdot \frac{1 \cancel{\text{hr}}}{3600 \text{ s}} \cdot \frac{5280 \text{ ft}}{1 \cancel{\text{ml}}} \\
&= \left(60 \times \frac{5280}{3600}\right) \text{ ft/s} \\
&= 88 \text{ ft/s.}
\end{aligned}$$

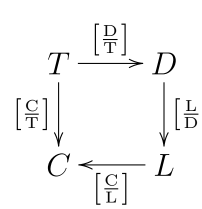
8. Example. On the planet Klingondo the following bizarre system of currency is used

1 Tiburón	=	5 Dorados
3 Dorados	=	20 Langostas
2 Langostas	=	5 Camarones

   Convert 6 T (Tiburones) to C (Camarones).

   Solution: We make the 3 successive changes of unit indicated in the diagram; first from T to D, next from D to L and finally from L to C. The conversion factor from T to C is given by

$$\begin{aligned}
\left[\frac{\mathrm{C}}{\mathrm{T}}\right] &= \left[\frac{\mathrm{D}}{\mathrm{T}}\right] \cdot \left[\frac{\mathrm{L}}{\mathrm{D}}\right] \cdot \left[\frac{\mathrm{C}}{\mathrm{L}}\right] \\
&= 5 \cdot \frac{20}{3} \cdot \frac{5}{2} \\
&= \frac{500}{6}
\end{aligned}$$

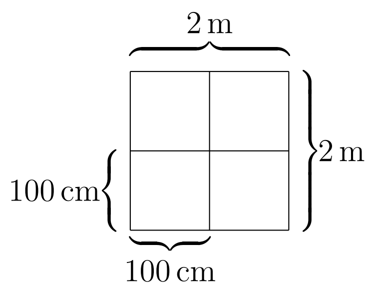


Thus

$$\begin{aligned} 6 \text{ Tiburones} &= \left(6 \times \frac{500}{6}\right) \text{ Camarones} \\ &= 500\,\text{Camarones}. \end{aligned}$$

9. Conversion of area units.

   Example: A square of dimensions 2 m× 2 m has area (2 × 2) m^2 = 4 m^2. The conversion factor from meters to centimeters $\left[\frac{\text{cm}}{\text{m}}\right] = 100$. Thus the sides of the square each has measure 200 cm.

   The area of the square in units of cm^2 is thus 200 cm × 200 cm = (4 × 10000) cm^2. Notice that the conversion factor from m^2 to cm^2 is the square of the conversion factor from m to cm, i.e.,

   $$\left[\frac{\text{cm}^2}{\text{m}^2}\right] = \left[\frac{\text{cm}}{\text{m}}\right]^2$$

   In general, if x and y are units of length, the conversion factor for area from units y^2 to units x^2 is given by

   $$\left[\frac{\text{x}^2}{\text{y}^2}\right] = \left[\frac{\text{x}}{\text{y}}\right]^2.$$

10. This fact has ramifications not only for squares. If we triple the radius of a circular pizza, the area changes by a factor 3^2 and not by 3 –as most people believe. More generally, a dilatation of the plane by a factor $c > 0$ produces a change in area of c^2.

11. Example: If there are 2.54 cm in 1 inch, how many mm^2 are there in $8\,\text{in}^2$?

    $$\begin{aligned} \left[\frac{\text{mm}^2}{\text{in}^2}\right] &= \left[\frac{\text{cm}^2}{\text{in}^2}\right] \cdot \left[\frac{\text{mm}^2}{\text{cm}^2}\right] \\ &= \left[\frac{\text{cm}}{\text{in}}\right]^2 \cdot \left[\frac{\text{mm}}{\text{cm}}\right]^2 \\ &= (2.54)^2 \times (10)^2. \end{aligned}$$

    Thus

    $$\begin{aligned} 8\,\text{in}^2 &= (8 \times (2.54)^2 \times 10^2)\,\text{mm}^2 \\ &= 5.16 \times 10^3\,\text{mm}^2. \end{aligned}$$

    As mentioned previously, chemistry texts would undoubtedly invoke the "Law of Thoughtless Cancellation" and display the above calculation as follows:

    $$\begin{aligned} 8\,\text{in}^2 &= 8\,\cancel{\text{in}}^2 \times \left(2.54\,\frac{\cancel{\text{cm}}}{\cancel{\text{in}}}\right)^2 \times \left(10\,\frac{\text{mm}}{\cancel{\text{cm}}}\right)^2 \\ &= 8 \times (2.54)^2 \times (10)^2\,\text{mm}^2 \\ &= 5.16 \times 10^3\,\text{mm}^2. \end{aligned}$$