

Useful Valid Arguments

1. Double Negation

$$\frac{\neg\neg p}{p}$$

2. Modus Ponens

$$\frac{p \rightarrow q}{p}$$

$$q$$

3. Modus Tollens

$$\frac{p \rightarrow q}{\neg q}$$

$$\neg p$$

4. Absurdity (Contradiction)

$$\frac{p \rightarrow (q \wedge \neg q)}{\neg p}$$

5. Addition Laws

$$(a) \frac{p}{p \vee q}$$

$$(b) \frac{p}{p \wedge (q \vee \neg q)}$$

6. Simplification Laws

$$(a) \frac{p \wedge q}{p}$$

$$(b) \frac{p \wedge (q \vee \neg q)}{p}$$

7. De Morgan's Laws

$$(a) \frac{\neg(p \wedge q)}{\neg p \vee \neg q}$$

$$(b) \frac{\neg(p \vee q)}{\neg p \wedge \neg q}$$

8. Contraposition

$$\frac{\neg q \rightarrow \neg p}{p \rightarrow q}$$

9. Syllogism

$$\frac{p \rightarrow q}{q \rightarrow r}$$

$$p \rightarrow q$$

10. More Arguments Involving Implication

$$(a) \frac{\neg(p \wedge \neg q)}{p \rightarrow q}$$

$$(b) \frac{p \rightarrow q}{q \rightarrow p}$$

$$p \leftrightarrow q$$

$$(c) \frac{p \rightarrow q}{\neg p \rightarrow \neg q}$$

$$p \leftrightarrow q$$

11. Distributive Laws

$$(a) \frac{p \wedge (q \vee r)}{(p \wedge q) \vee (p \wedge r)}$$

$$(b) \frac{p \vee (q \wedge r)}{(p \vee q) \wedge (p \vee r)}$$

12. Commutative Laws

$$(a) \frac{p \vee q}{q \vee p}$$

$$(b) \frac{p \wedge q}{q \wedge p}$$

13. Associative Laws

$$(a) \frac{(p \vee q) \vee r}{p \vee (q \vee r)}$$

$$(b) \frac{(p \wedge q) \wedge r}{p \wedge (q \wedge r)}$$